Crystal Motional Parameters
A Comparison of Measurement Approaches

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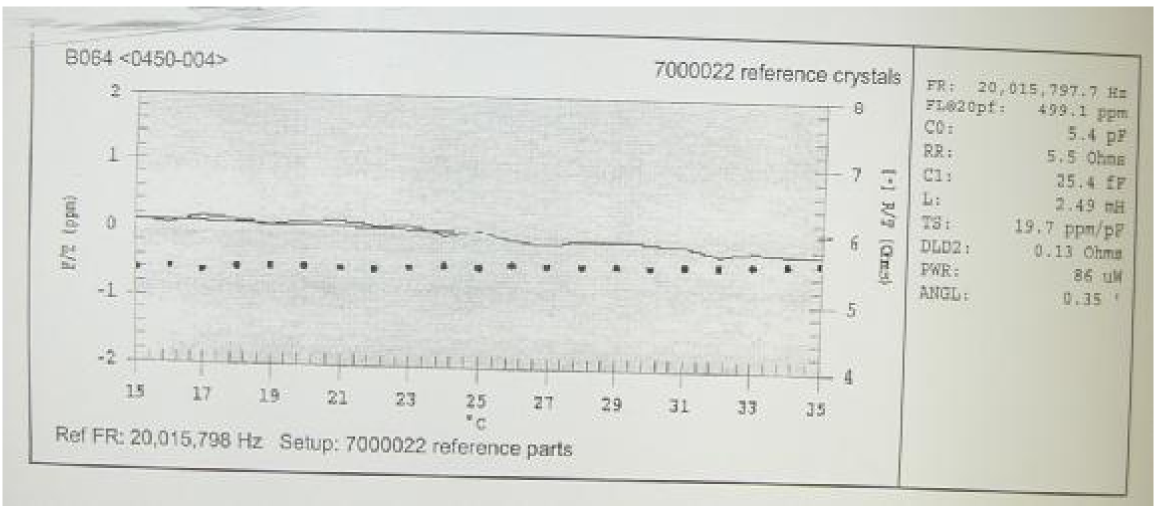
11 June 2006

This paper compares results of a variety of alternative techniques to measure the motional parameters of a quartz crystal.

The measured crystal is a Saunders & Associates Reference Crystal, model 7000022, with the following parameters, as characterized with an S&A 250B automated parameter test set:



The key parameters for our purpose are:

C0: Static (holder) capacitance
C1: Motional capacitance
L: Motional inductance
RR: Motional resistance

In this study, the motional parameters will be identified with the subscript M, e.g., C_M.

The reference crystal has the following features:

- 20 MHz AT cut fundamental mode crystal
- Low aging design (<0.5ppm/year)
- Zero temperature coefficient at room temperature

All errors are calculated with respect to the parameters provided by S&A.

The following table summarizes the results of 18 measurements:

Method	Test Number	Fixture	Methodology	Co (pF)	C1 (fF)	L1 (mH)	R1 (ohms)	C1 Error %	R1 Error %
Saunders & Associates Calibration Standard	0	N/A	N/A	5.4	25.4	2.49	5.5	Std	Std
Boonton 250RX Meter C0 only @ 1 MHz	1	N/A	N/A	5.52					
G3UUR	2	N/A	1	--	24.73	2.65	--	-2.6%	--
09 Jun Pi Fixture & HP8752B	3	a	2	5.46	25.987	2.433	6.77	2.3%	23.1%
50 Ohm Transformer Fixture & HP8752B	4	d	2	--	28.91	--	6.5275	13.8%	18.7%
Pi fixture SN-002 remeasure w/ HP8752B	5	a	2	--	26.446	--	6.392	4.1%	16.2%
87510A with Fixture SN-001, 300 KHz Span, AUTO mode	6	a	5	--	25.883	2.4475	6.6023	1.9%	20.0%
87510A with Fixture SN-001, 10 KHz span, power out: +20 dBm	7	a	5	--	26.463	2.3892	6.65	4.2%	20.9%
87510A with Fixture SN-001, 10 KHz span, power out: +10 dBm	8	a	5	--	26.468	2.3888	6.644	4.2%	20.8%
87510A with Fixture SN-001, 10 KHz span, power out: 0 dBm	9	a	5	--	26.455	2.39	6.665	4.2%	21.2%
87510A with Fixture SN-001, 10 KHz span, power out: -10 dBm	10	a	5	--	26.479	2.3889	6.659	4.2%	21.1%
Fixture SN-002, HP8752B, +/- 45 degree method	11	a	3	--	26.26	--	6.659	3.4%	21.1%
Remeasure 50 ohm transformer fixture, HP8752B	12	d	3	--	28.872	--	3.72	13.7%	-32.4%
12.5 Ohm Xfmr Fixture, HP8752B	13	b	3		50.55	--	2.719	99.0%	-50.6%
50 ohm die-cast fixture, with 20 dB (in) & 6 dB pads (out), 0 dBm out of HP8752B, +/- 45 degree phase method	14	c	3		25.792		6.2916	1.5%	14.4%
Same setup as 14, except measure fr & fs and calculate based on C0; shunting C only C0 and stray fixture	15	c	3	--	28.65	--	--	12.8%	--
Same setup as 15, but with known 10.11 pF shunting capacitor	16	c	3	--	28.50	--	5.83	12.2%	5.9%
HP8752B, 10 dB pad, Pi fixture sn-002, resistance via substitution method, Lm via Q measurement	17	a	3	--	26.15	2.42	6.18	2.9%	12.3%
Resistance only via reflection, HP8752B	18	f	6				6.82	--	24.1%

Fixture	**a**	12.5 ohm resistive pi
	b	12.5 ohm transformer
	c	50 ohm - no transformer
	d	50 ohm transformer
	e	Reflection holder
Methodology	**1**	G3UUR Oscillator frequency shift
	2	+/- 45 degree phase shift
	3	-3 dB bandwidth
	4	series and parallel resonance
	5	HP87510A Automatic characterization
	6	Reflection measurement

The methodologies and test fixtures are described in more detail later in this document.

The following test equipment was used in these tests:

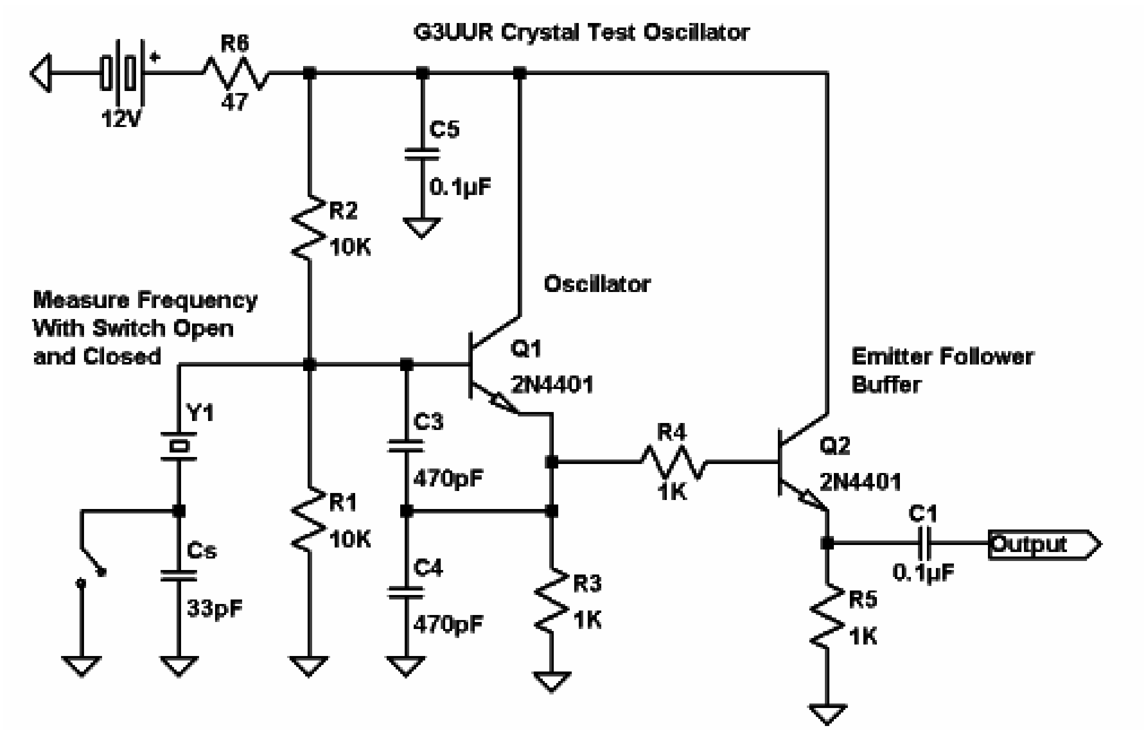
HP8752B network analyzer, 300 KHz-3 GHz, with Minicircuits model UNMP-5075 75 ohm : 50 ohm matching pads
HP87510A gain/phase analyzer, 100 KHz – 300 MHz
Miscellaneous Minicircuit 50-ohm standards and pads, and miscellaneous adapters and cables.
Boonton RX meter Type 250A
HP 6205C DC power supply
Heath IM-2420 frequency counter
BP Model DCM-601 digital capacitance meter

Homebrew G3UUR oscillator
Homebrew test fixtures as described.

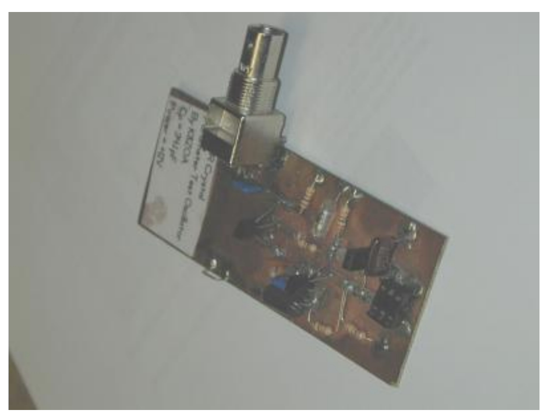
Part I – Measurement Methodologies

1. G3UUR Oscillator – Frequency Shift Method

This method was developed by David Gordon-Smith, G3UUR, and popularized by Wes Hayward, W7ZOI. [1]



The above schematic shows the circuit I built. It follows the design of Reference 1, substituting 2N4401s for the 2N3904s in the original design. I used Manhattan-style construction to build my unit.



The procedure is to measure the frequency with the switch open and closed. When open, the crystal is in series with C2 and C0, the crystal's holder capacitance.

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1 Hayward, Wes, Campbell, Rick, and Larkin, Bob, "Experimental Methods in RF Design," (2003) American Radio Relay League, Hartford, CT at Page 3.19.

To calculate the motional parameters, use the following equations:

$$C_M = \frac{2(C_S + C_0)\Delta f}{f}$$

$$L_M = \frac{1}{(2\pi f)^2 C_M}$$

Cs should also include the strays associated with the switch and wiring. I measured Cs=34.1 pF in my oscillator. (To reduce stray capacitance, I used a two-pin header and shunt jumper instead of a standard switch.)

C0 is the crystal holder capacitance.

Δf is the shift in frequency between switch open and switch closed

f is the frequency with switch closed.

Remember to use proper units; capacitance is in farads (1pF = 10^{-12} Farads), f and Δf are in Hz, Lm is in Henries.

Sample calculation with S&A calibrated crystal:

Measured Parameters
Switch closed: 20016843 Hz
Switch open: 20023109 Hz
Δf = 6266 Hz
C0 = 5.52 pF = 5.52*10^{-12} F (measured with Boonton model 250A RX meter at 1 MHz)
Cs = 34.1 pF = 34.1*10^{-12} F (measured with BP model DCM-601 digital capacitance meter)

Calculated Parameters

$$C_M = \frac{2\times(5.52\times10^{-12} + 34.1\times10^{-12})\times 6266}{20016843} = 24.73\times10^{-15}\,F = 24.73\,fF$$

$$L_M = \frac{1}{(2\times\pi\times 20016843)^2 \times 24.73\times10^{-15}} = 2.56\times10^{-3}\,H = 2.56\,mH$$

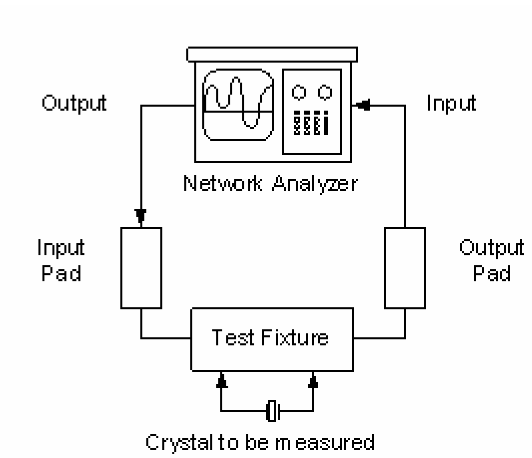
A useful unit for motional capacitance is the femto farad, where 1000 fF = 1pF.

We may compare the measured parameters with the calibration information:

Parameter	Measured	Calibration Standard	Error
Cm	24.73 fF	25.4 fF	-2.6%
Lm	2.56 mH	2.49 mH	+2.8%
C0	5.52 pF	5.4 pF	+2.2%
Rm	Not measured	5.5 ohms	N/A

2. Phase Shift Measurement

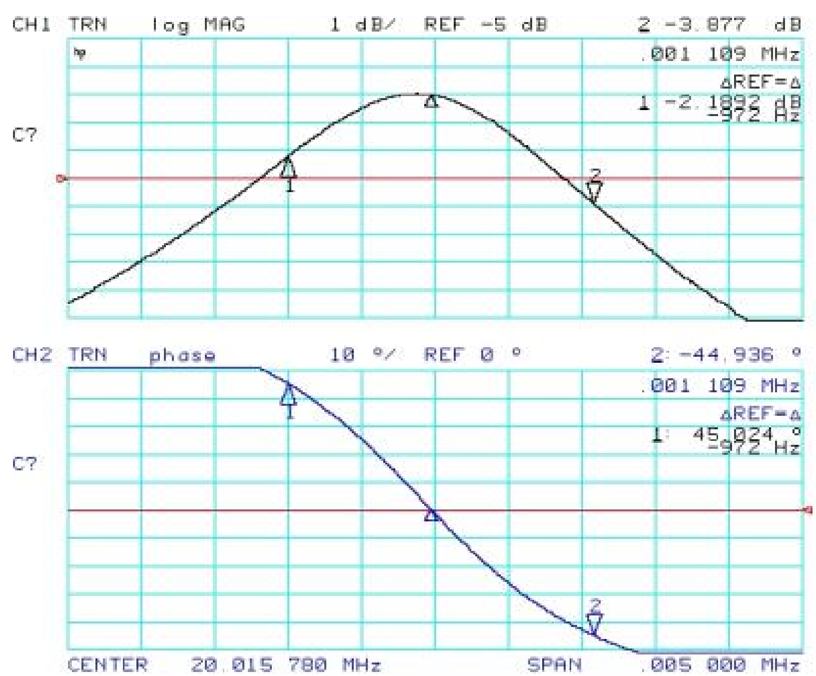
The method recommended in international standards is to install the crystal in a low impedance test fixture, fed with an accurate, stable signal generator and measure the phase shift of the transmission signal near resonance.



Historically, a stable signal generator and a vector voltmeter have been used to measure the phase of the transmission signal. In my case, I used either an HP8752B network analyzer, or an HP87510A gain/phase analyzer to generate the signal and measure the amplitude and phase of the transmission signal.

Methodology and Equations

The figure to the right shows the amplitude (top panel) and phase (bottom panel) of the S&A calibration crystal in Fixture (a), the 12.5 ohm resistive pi design



The bottom trace has markers at the +45 and –45 degree points.

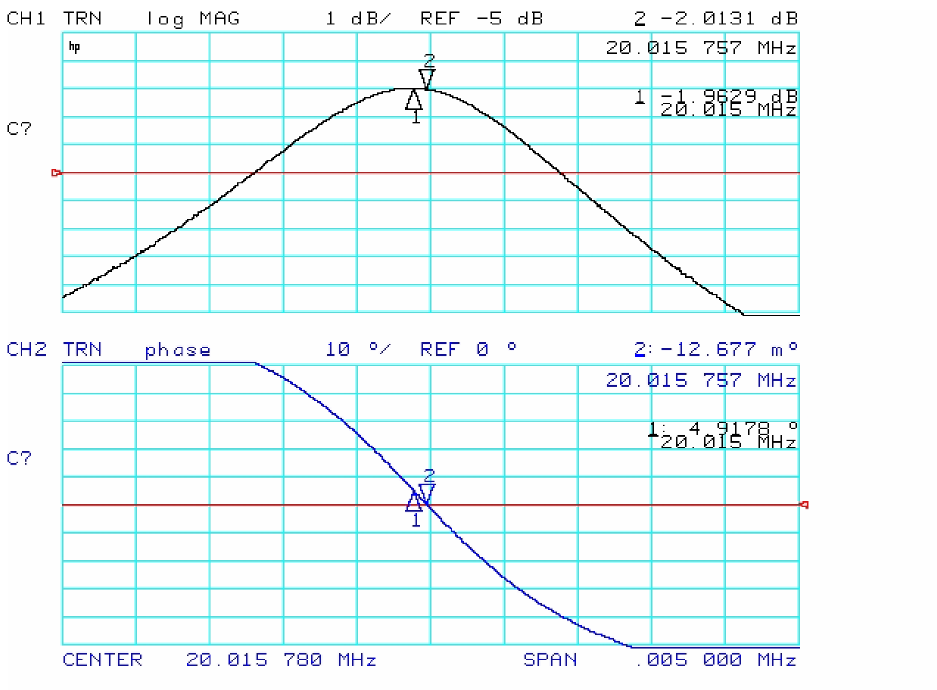
The +45 degree point is at –972 Hz below the zero angle reference, whilst the –45 degree point is 1109 Hz above the zero reference.

The ±45 degree bandwidth is thus 2081 Hz.

We also use the same setup to measure Rm, by measuring the insertion loss at the zero phase point.

When phase = 0 (marker 2), the insertion loss is 2.01131 dB.

Note that the zero phase point is not the minimum loss point. In fact, the two are 89 Hz separated in this measurement.



The relationship between attenuation and series resistance is:

$$R_M = 2R_L\left(10^{\frac{\alpha}{20}} - 1\right)$$

Where

R_M is the motional resistance
R_L is the source and load resistance seen by the crystal (a function of the test fixture)
α is the loss in dB

Applying the measured data, we calculate R_M:

$$R_M = 2 \times 12.5 \times \left(10^{\frac{2.0131}{20}} - 1\right) = 6.521\Omega$$

The total resistance, R_{EFF}, seen by the crystal is the sum of the load resistance (input and output) and the motional resistance, R_M:

$R_{EFF} = 2\ R_L + R_M = 2 * 12.5 * 6.52 = 31.52$ Ohms.

The following formulas may be used to compute the motional parameters.

$$C_M = \frac{\Delta f\,(\pm\vartheta)}{2\pi {f_R}^2 R_{EFF}\,\tan(\vartheta)}$$

$$L_M = \frac{R_{EFF}}{2\pi\Delta f(\pm\varphi)}\tan(\vartheta)$$

The standard calls for measurement at the ±45°, so φ = 45°

Where

Δf is the frequency diffrerence between the ±45° points, measured at 2081 Hz.
φ = 45°, so tan(45) = 1.
R_{EFF} = 31.52 ohms, as determined above
f_R is the series resonant frequency, *i.e.,* the frequency at which φ=0°.

We now calculate the motional parameters:

$$C_M = \frac{2081}{2\times\pi\times 20015757^2\times 31.52\times 1} = 26.23\times 10^{-15}F = 26.23\,fF$$

$$L_M = \frac{31.52}{2\times\pi\times 2081}\times 1 = 2.411\times 10^{-3}H = 2.411mH$$

We now compare these measured values with the calibration standard:

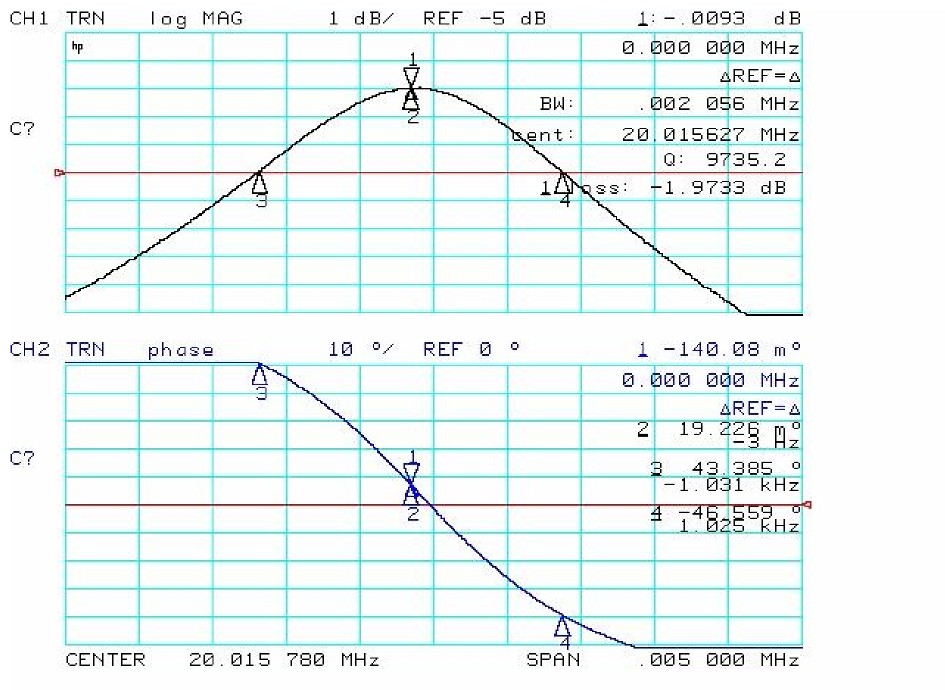
Parameter	**Measured**	**Calibration Standard**	**Error**
Cm	26.23 fF	25.4 fF	+3.3%
Lm	2.411 mH	2.49 mH	-3.2%
C0	Not measured	5.4 pF	
Rm	6.52	5.5 ohms	+18.5%

3. -3 dB Bandwidth Method

Measuring the ±45° bandwidth requires equipment not easily available to all experimenters.[2]

It's also possible to derive motional parameters from a 3 dB bandwidth measurement. This may be made with much simpler equipment.

The figure at the right shows the -3dB bandwidth data for the calibration crystal. The crystal is installed in the type (a) 12.5 ohm resistive pi fixture.



The network analyzer calculates the 3 dB bandwidth as 2056 Hz, and the measured Q as 9737.5,with a center frequency of 20.015627 MHz. The attenuation at the point of minimum loss is 1.9733 dB.

We base the motional parameter calculation on the definition of Q:

$$Q = \frac{f_R}{\Delta f_{-3dB}} = \frac{2\pi f_r L_M}{R_{EFF}}$$

Solving for L_M we find:

$$L_M = \frac{QR_{EFF}}{2\pi f_R}$$

Where

R_{EFF} is the effective series resistance seen by the crystal, R_M + 2 * R_L.
f_R is the series resonant frequency, i.e., the frequency of minimum loss, 20.015627 MHz.
Δf is the –3dB bandwidth, displayed as 2056 Hz.
Q is the ratio of f_R to the –3dB bandwidth, displayed as 9737.5 (or can be calculated as 20015627 / 2056 = 9735)

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[2] See K8IQY's low-cost approach to 3 dB crystal parameter measurements at http://www.k8iqy.com/testequipment/pvxo/pvxopage.htm and http://www.k8iqy.com/testequipment/pvxo/Atlanticon2002V1R5.pdf.

$$C_M = \frac{1}{4\pi^2 f_R^2 L_M}$$

Using the methodology described before, we calculate R_M and R_{EFF} using the measured 1.9733 dB loss at the center frequency.

R_M = 6.38 ohms, REFF = 31.38 ohms.

Calculating L_M:

$$L_M = \frac{9737.5 \times 31.38}{2 \times \pi \times 20.015627 \times 10^6} = 2.429 \times 10^{-3} H = 2.429 mH$$

$$C_M = \frac{1}{4 \times \pi^2 \times 20015627^2 \times 2.429 \times 10^{-3}} = 26.03 \times 10^{-15} F = 26.03 fF$$

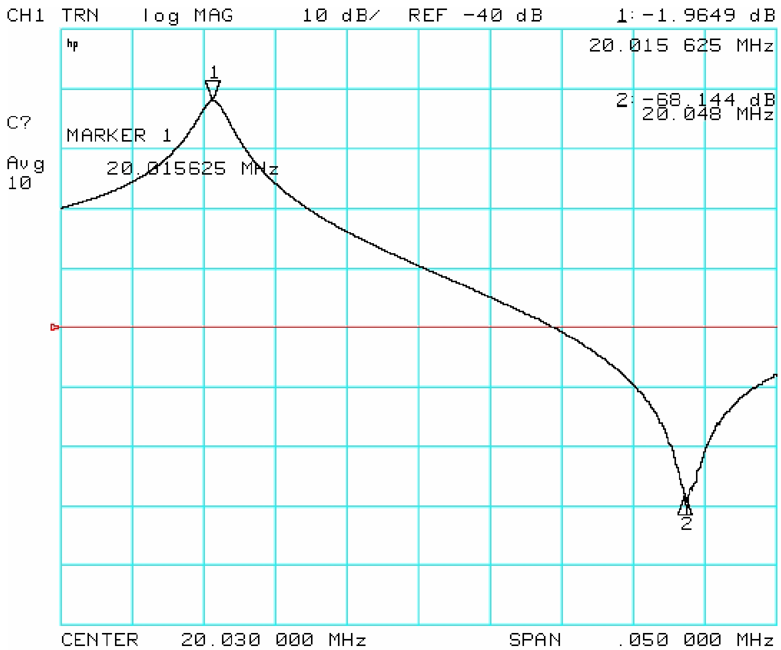
We now compare these measured values with the calibration standard:

Parameter	**Measured**	**Calibration Standard**	**Error**
Cm	26.03 fF	25.4 fF	+2.5%
Lm	2.429 mH	2.49 mH	-2.4%
C0	Not measured	5.4 pF	
Rm	6.38	5.5 ohms	+16.0%

4. Series and Parallel Resonant Frequencies

A variant on the two frequency method used in the G3UUR oscillator is to measure the series and parallel resonant frequencies of the crystal, and from their ratio determine the motional parameters.

The figure at the right shows the series f_S (at marker 1) and parallel f_P (marker 2) resonant frequencies of the calibrated crystal.



The precise frequencies measured are:

MARKER PARAMETERS	Channel 1
MARKER 1	20.015625 MHz -1.9649 dB
MARKER 2	20.048625 MHz -68.144 dB

The relationship between motional parameters and the series and parallel resonant frequencies is:

$$C_M = \left(\frac{f_P}{f_S} - 1\right) 2(C_0 + C_{STRAY})$$

$$L_M = \frac{1}{4\pi^2 f_R^2 C_M}$$

Where

f_P and f_S are the parallel and series resonant frequencies as described above
C0 is the holder capacitance
C_{STRAY} is the stray shunting capacitance of the test fixture

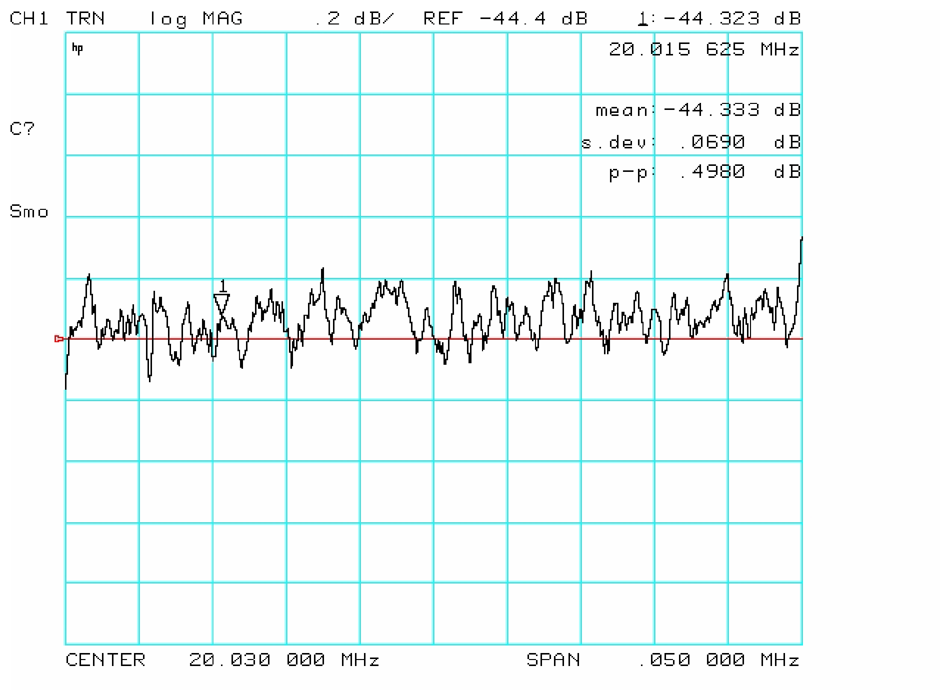
To measure C_{STRAY}, we determine the attenuation through the test fixture with the crystal removed.

Assuming that the stray capacitance is small, we can use the attenuation to determine the corresponding capacitive reactance and from that the value of Cstray.

With the crystal removed, the attenuation has a mean value of 44.3 dB at 20.03 MHz.

This corresponds to a capacitive reactance of 4.08 K ohm, corresponding to1.95 pF stray capacitance shunting the crystal socket.

As previously mentioned, measuring the calibrated crystal at 1.0 MHz with a Boonton 250 RX meter showed the C0 = 5.61 pF.



Hence, $C0 + C_{STRAY} = 5.52 + 1.95$ pF = 7.47 pF.

We now are in a position to calculate the motional parameters:

$$C_M = \left(\frac{20048625}{20015625} - 1\right) \times 2 \times 7.47\, pF = 24.63\, fF$$

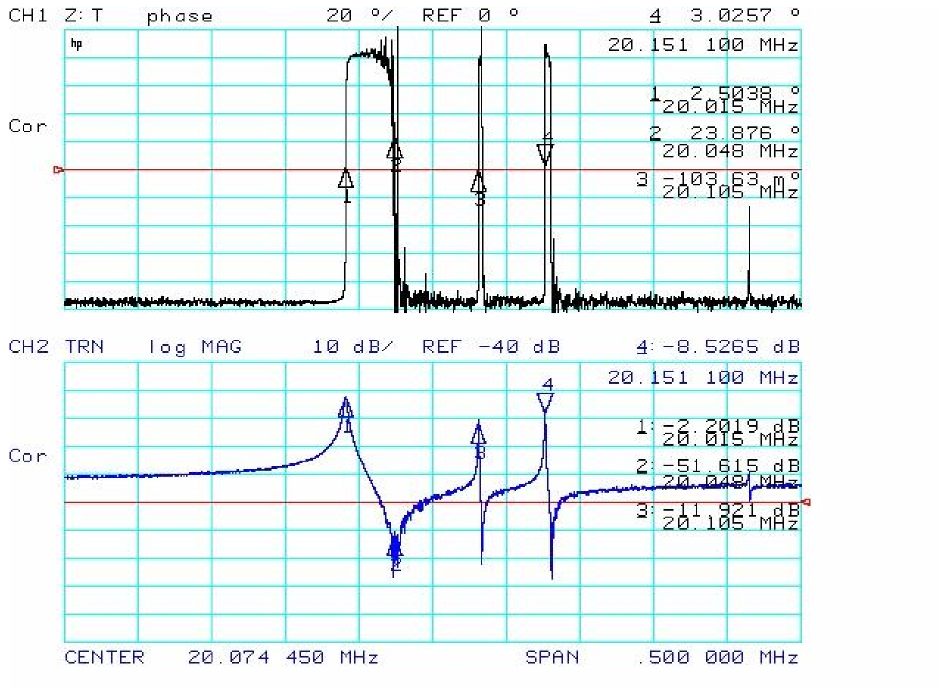
$$L_M = \frac{1}{4 \times \pi^2 \times 20015625^2 \times 24.63 \times 10^{-15}} = 2.57\, mH$$

We now compare these measured values with the calibration standard:

Parameter	Measured	Calibration Standard	Error
Cm	24.63 fF	25.4 fF	-3.0%
Lm	2.57 mH	2.49 mH	+3.2%
C0	5.61	5.4 pF	+3.9%
Rm	6.35	5.5 ohms	+15.4%

(Rm is computed from the marker 1 attenuation value of 1,9649 dB)

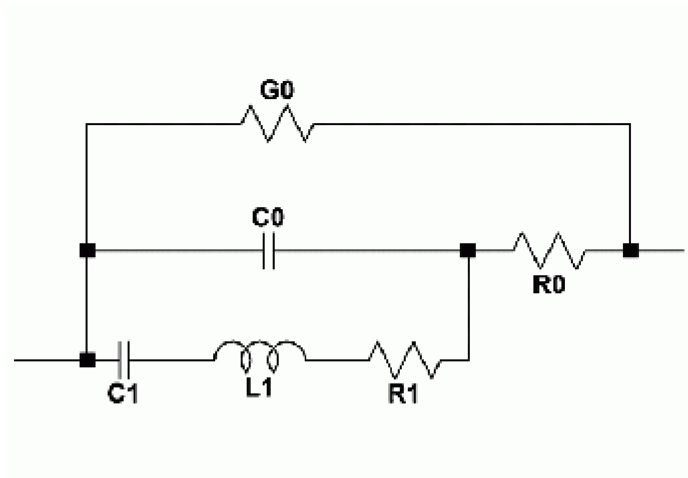
When measuring the series and parallel resonant frequencies, be sure you don't inadvertently measure a spurious resonance, as most crystals exhibit several spurious resonance points above the true parallel resonance point.



The calibrated crystal, for example, shows two major and one minor spurious resonance points.

5. HP87510A Automatic characterization

The HP87510A gain/phase analyzer has an internal firmware crystal resonator characterization routine that computes and displays a six-parameter model. To use the automatic characterization function, the crystal must be installed in a transmission fixture.



The primary advantage of the 87510A's automatic characterization function is that its analysis is based upon curve fitted data involving at least 8 measurement points. It thus offers the prospect of increased accuracy by leveraging multiple data points.

The 87510A's Operation Manual describes the methodology used to calculate the six parameter values:

1. Obtains the admittance characteristic circle diagram
2. Obtains the maximum conductance (G_{max})
3. Obtains frequencies f_1 and f_2 ($f_1<f_2$) of two points where conductance is half the maximum conductance.
4. Calculate f_s by $f_s = \sqrt{f_1 \times f_2}$
5. Obtains susceptance B_{fs} at f_s
6. Calculate ω_S by $\omega_S = 2\pi\ f_S$
7. Assumes that the frequency at which the phase difference becomes 0° near the parallel resonance frequency is f_a and obtains its conductance G_a.
8. Calculate ω_a by $\omega_a = 2\pi\ f_a$
9. Assumes that the frequency at which the phase difference becomes 0° near the series resonance frequency is f_r.
10. Calculates the constants using the above values and the following equations:

$$Q_S = \frac{f_S}{f_2 - f_1} \qquad C_0^{'} = \frac{B_1 + B_2}{2\omega_S}$$

$$L_1 = \frac{Q_S}{\omega_S G_{max}}$$

$$C_1 = \frac{G_{max}}{\omega_S Q_S}$$

$$C_0 = \frac{B_{fs}}{\omega_S}$$

$$R_1 = \frac{C_0^{'}}{C_0 G_{max}}$$

$$R_0 = \frac{1}{G_{max}} - R_1$$

$$G_0 = G_a - \frac{R_1 \omega_a^2 C_0^2}{1 + R_0 R_1 \omega_a^2 C_0^2}$$

Applying these equations by pressing the 87510A's appropriate soft key produced the following motional parameters and errors:

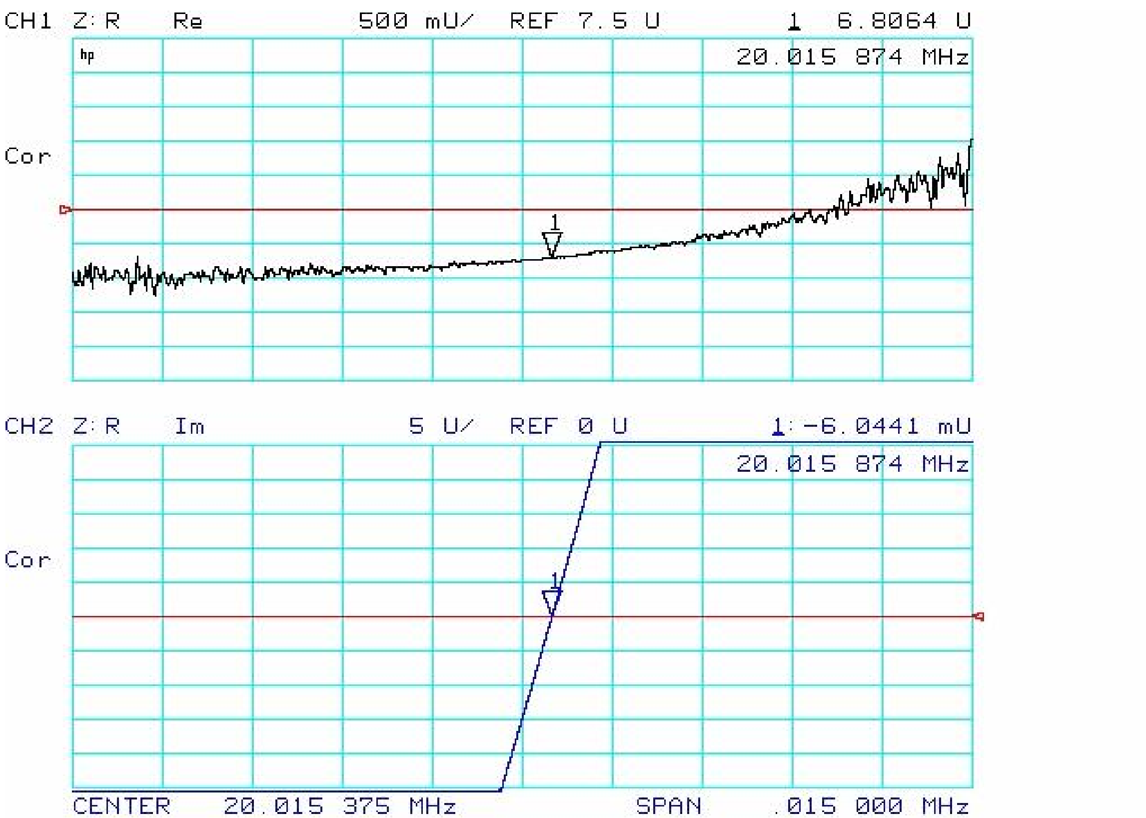
Parameter	**Measured**	**Calibration Standard**	**Error**
Cm	25.883 fF	25.4 fF	+1.9%
Lm	2.4475 mH	2.49 mH	-1.7%
C0	N/A	5.4 pF	
Rm	6.60	5.5 ohms	+20.0%

The measured parameters are sensitive to the span used and the above data represents the best case. Other 87510A measurements are summarized in the measurement table and display errors up to 4.2%.

6. Reflection Resistance Measurement

The final measurement approach was an attempt to resolve the wide gap between the reference crystal's stated motional resistance of 5.5 ohms, whilst all my measurements were in the range 6.3 to 6.8 ohms, with 6.6 ohms being the most common value.

In this mode, the network analyzer is successively calibrated, at the reflection measurement port, with an open, short and load (50 ohm) standard. The unknown is attached to the reflection measurement port and the instrument set to internally convert the complex reflection coefficient to either Z/theta or R and jX values.



The data confirms the transmission-mode values in the 6.5 ohm range. At the point where the imaginary (reactive) part of the impedance is zero, the real (resistive) value is 6.8 ohms.

Some crystals exhibit significant change in motional resistance as drive level changes. To test this as a possible reason for

the consistent divergence between measured and calibration motional resistance, I ran a power sweep test using the HP87510A gain/phase analyzer.

As shown, over the range -50 dBm to +20 dBm, R_M changes less than 0.04 ohms and stays in the 6.6 ohm range.

The crystal is held in a (a) type resistive fixture, with 10 dB attenuation between the 87510A's output and the fixture.

The extra attenuation, plus the fixture's loss, means that the power delivered to the crystal is much less than the 87510A's output shown on the horizontal axis. +20 dBm corresponds to approximately 4 mA crystal current, or about 100 microwatts.

Marker 1, at +19.26 dBm corresponds to 88 microwatts, the power level at which the calibrated crystal was tested by S&A. The measured value at this point is 6.62 ohms.

To verify the accuracy of the power sweep and test fixture, I substituted a 1% 10 ohm, 1206 surface mount resistor for the test crystal.

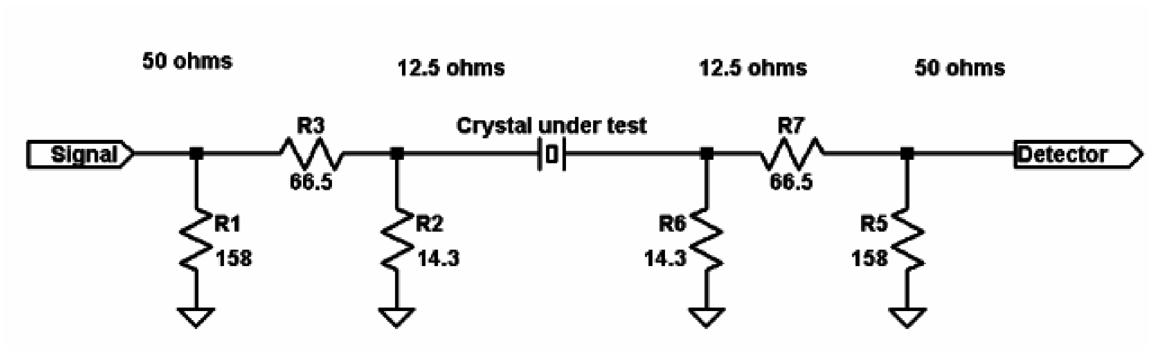
Although there is some variation, the 87510A results are quite close to the test resistor's 10.0 ohm market value.

At DC, a four-wire measurement shows the test resistor is 10.03 ohms.

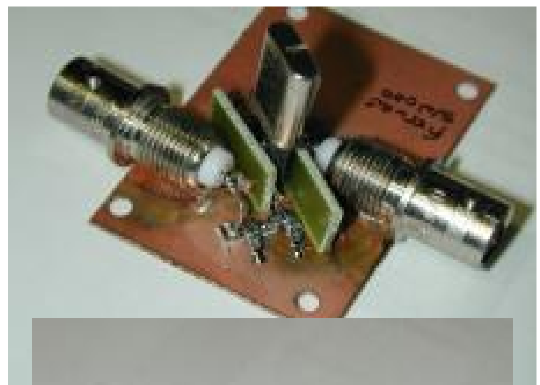
Part II – Test Fixtures

I built and used five test fixtures:

(a) Resistive Pi Fixture:

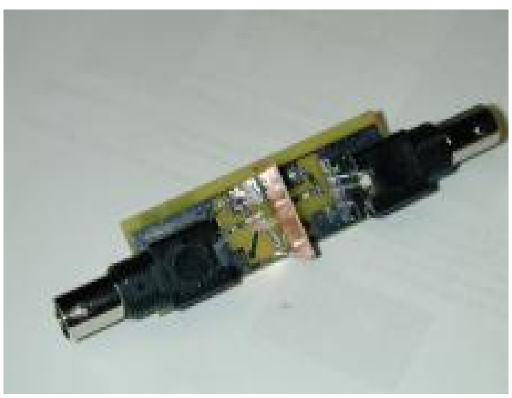


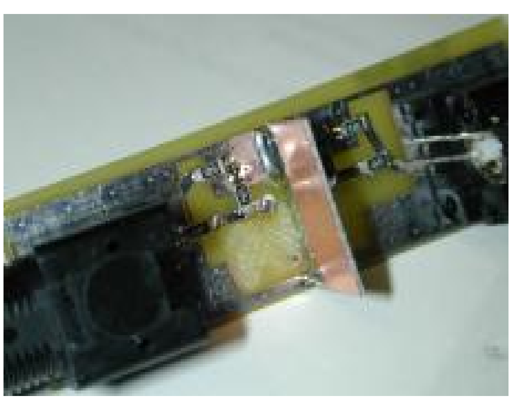
The resistive pi fixture matches the 50 ohm input and output to 12.5 ohms. The ones I built follow the standard design in IEC standard, except that the phase compensation trimmers across the 14.3 ohm resistors are not used and I used the nearest 1% standard resistor values.



The photo to the right shows the prototype pad unit I built, using Manhattan-style construction. The pad is constructed with re 1%, 1206 size surface mount resistors.

It's important to shield the input and output ports.



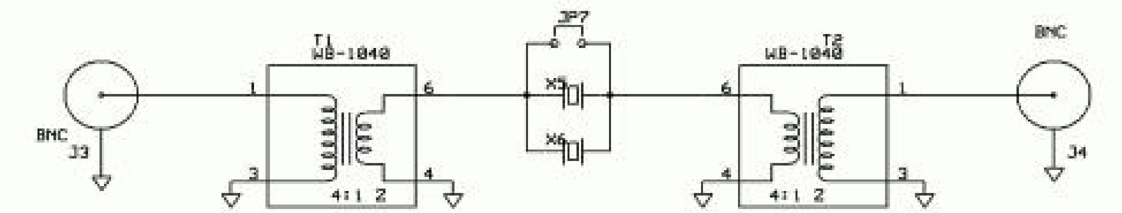


The three photos show the printed circuit version of the prototype.

The resistors provide a reasonably good impedance match and hence external stabilizing pads are less necessary than for other fixtures.

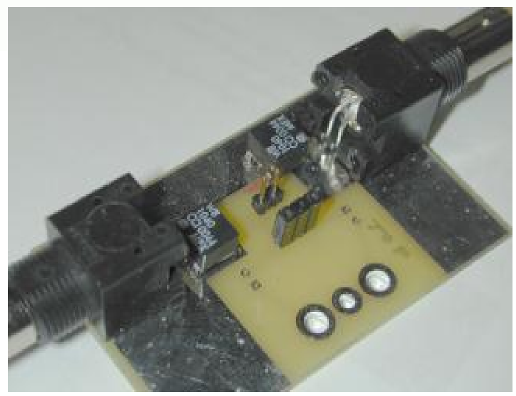
(b) Transformer Pi Fixture

In addition to a resistive pad matching approach, it's possible to use a 2:1 winding transformer to provide a 50ohm to 12.5 ohm impedance transformation.



My transformer fixture uses two Coilcraft WB1040 transformers. The transformers have a bandwidth of 200 KHz – 300 MHz.

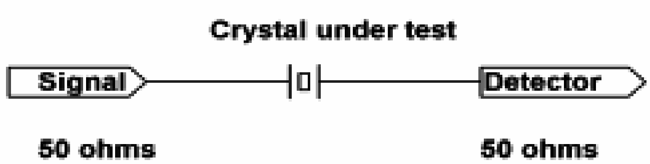
Any error in impedance on the 50 ohm side will be reflected in errors in the 12.5 ohm transformed side, so stabilizing pads will normally be employed on both sides of the fixture.



(c) 50 Ohms, no Matching, no Loss

The simplest fixture simply breaks the connection between signal generator and receiver and inserts the crystal.

I built one into a Hammond die-cast box and brought the connections out to binding posts.

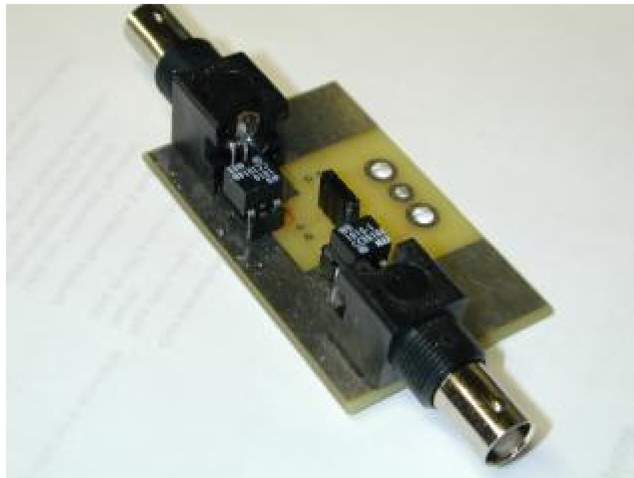


In order to accurately compute the crystal motional parameters, it's important that the impedance seen by the crystal be accurately known. With a simple fixture of this type, it's important to use pads on both the input and output in order to provide accurate and stable impedance.

(d) 50 Ohm Transformer Fixture

I also built a variant of the 12.5 ohm transformer fixture described at (b), but with Coilcraft model WB1010 transformers providing a 1:1 impedance ratio.

It is otherwise identical with the fixture discussed in section (a). This fixture is also to be used only with a controlled impedance environment, as the transformers reflect actual input and output impedance.



(e) Reflection Test Fixture

To measure R_M via reflection, the reference crystal was attached to the 8752B network analyzer using a standard BNC-to-binding post adapter, HP model 10110A.